**Cαpital Risk**
Quantitative Research

December 2017

Equity Markets

# A Primer on Multi-Factor Models

- Global equity markets are increasingly integrated.
- Country factors may not provide optimal diversification.
- Diversification by global factors can manage portfolio risk.
- Portfolio factor tilts can express specific views on return factors.

Multi-factors models are a leading portfolio management tool due to the pioneering work of Fama and French (1992) who showed that more than a single factor explained US equity market returns. Investors enhance portfolio value by either increasing return or reducing risk (as measured by variance in the time series of returns) to achieve portfolio efficiency (Markowitz, 1952). The growing integration of global markets suggests that country factors may not be the singular driver of global returns and risk (Hau, 2011). Global factors (e.g. crude oil, commodities, etc.) may provide a more fulsome picture of the return and risk drivers in the global markets.

Some countries have exposure to crude oil, while some are significant exporters and thus have foreign exchange risk (Diermeier & Solnik, 2001). This situation does not imply that country (or sectors) do not have distinctive features that are unique to their return and risk profiles. It does suggest that the global investor should be aware of their portfolio return and risk profile to better manage their portfolio.

When entering global markets and investor asks the following questions:

1. What are the global factors that drive the risk across markets?
2. What are the global factors that drive returns across markets?
3. Are there rewards for exposure to these global factors?
4. Does isolating the global factors improve portfolio efficiency?

A model is required to answer these questions to measure the return and risk factors and, critically, to design a portfolio.

## Factor Models

While there are many models, the version deployed here is the Arbitrage Pricing Theory (APT) of Ross (1976), which uses a multivariate regression model.[1] When employing APT, the model makes three material assumptions:

- Multiple risk factors that explain risk and return
- These global risk factors define the systematic global risk
- The factors applied are a valid proxy for the actual risk structure

Using the model requires a necessary clarification. While a defined group of risk factors may drive risk, this group of risk factors may not necessarily determine the long-term returns of the markets. For example, changes in the prices of US dollar may lead to variation in the price of equity markets, but they need not be a determinant of the long-term returns for the same markets.

The global factors that deliver value in the long-term may be a subset of these same risk factors. While the exposure to the US dollar may have a long-term expected return of zero, the exposure to the equity markets may have a premium attached that compensates the investor for bearing the risk.

The process for factor models is in three parts:

1. A multifactor model to describe the return exposure
2. A variance decomposition to determine the risk exposure
3. Portfolio factor tilts

This process enables the investor to model both sides of the risk and return trade-off while providing the means to express a factor view within the portfolio.

## The Market Model

The market model computes the variances and covariance of returns to a market factor and is a staple of Modern Portfolio Theory (see: Fama, 1965; Lintner, 1965; Sharpe, 1964; Treynor, 1962). The use of one factor to explain all the variation simplifies the calculation process. This model separates the risk into two components: the systematic (undiversifiable) market risk and the unsystematic (idiosyncratic) risk.

[1] The model uses an unconditional sample and thus assume that the sample is the total population and will be indicative of the future.

For a given asset A, the return is modeled as:

$$R_A = \alpha_A + \beta_A R_M + \varepsilon_A$$

where: $R_A$ = Return of Asset A $R_M$ = Return of the Market
$\alpha_A$ = Alpha of Asset A $\varepsilon_A$ = error term
$\beta_{A,M}$ = Beta of Return of A to Return of the Market

This model makes three assumptions to ensure its validity. These include:

- Error term expected value = 0
- Market Return is uncorrelated with the error term
- Error terms of all assets are uncorrelated

These assumptions are similar to linear regression models, but with two fundamental differences: there is no assumption of a normally distributed error term, and the variances of the error terms are not necessarily equal. This structure implies that each asset can have specific risk.

This model leads to the following conclusions about the expected asset return:

- The return on the asset is relative to the market
- The variance of the asset is relative to the market
- Covariance of any two assets depends upon the variation of the market

The market model and its assumptions make the task of forecasting expected returns computationally less intensive but leave two significant burdens: a reliable forecast of the market variable, and the presumption of unconditional (stationary or unchanging) covariance of each asset with the market.

## Multi-Factor Models

The critical limitation of the market model is that it assumes one factor, the market index, explains *all* of the variations in any given asset. Given this limiting assumption, multifactor models provide two benefits: they offer the possibility to better define the behavior of an asset by including other factors; and two, enable a more robust explanation of the risk of an asset.

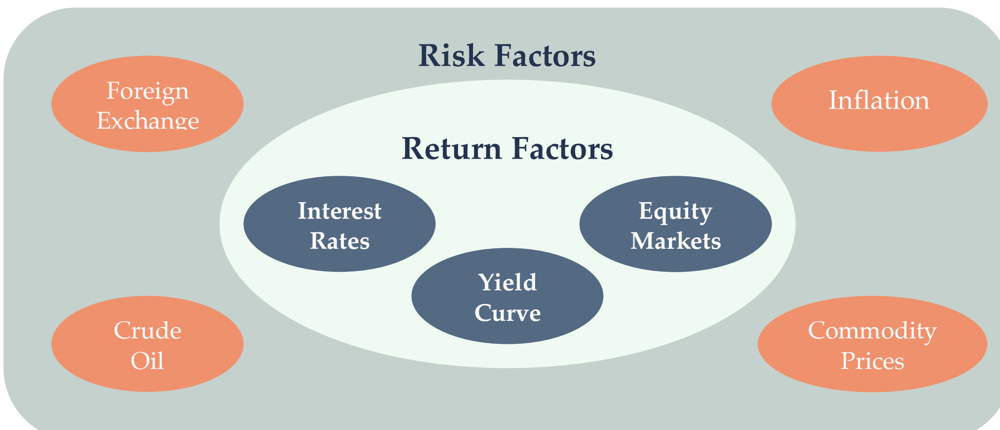
A traditional multifactor model for returns is specified as follows:

$$R_i = \alpha_i + \beta_{i1} F_1 + \beta_{i2} F_2 + \ldots + \beta_{iK} F_K + \varepsilon_i$$

where: $R_i$ = Return of Asset i
$\alpha_0$ = Expected Return of Asset i
$F_K$ = Factor K, where K = 1 to K
$\beta_{i1}$ = Beta of Return of Asset i to Factor K
$\varepsilon_i$ = error term

In this example, it is the regression of the portfolio return of the portfolio against the global factors. In Figure 1, the universe of factors is separated into risk and return elements. While all factors contribute to risk, it is only the return factors that provide a *long-term* expected return different from zero.

### Figure 1. Risk and Return Factors




One of the tenets of modern finance is that the markets are informationally efficient and that all public information is priced into the market (see: Cootner, 1962; Samuelson, 1965). The market price reflects the consensus of the participants and is the *average* view of the given factor. For an investor to achieve excess returns from a factor requires appropriately positioning the portfolio for *unexpected* events (i.e. events that the market is not pricing).

If the investor wanted to measure the impact on inflation on the portfolio returns, the recent release of inflation data that was at the consensus view of 2% is not of interest. The investor is interested in the release when it deviates from the consensus view, e.g. 1% above the expected level. Market participants evaluate the news and revalue their inflation positions due to the unexpected inflation. To more precisely measure the impact of these unexpected events, the investor adjusts their multi-factor model slightly.

As before the model is specified as follows, but with adjustment to the factor time series, so that they show the value not priced by the market:

$$R_i = \alpha_i + \beta_{i1} F_1 + \beta_{i2} F_2 + \ldots + \beta_{iK} F_K + \varepsilon_i$$

where: $R_i$ = Return of Asset i
$\alpha_0$ = Expected Return of Asset i
$F_K$ = Unexpected value of Factor K, where K = 1 to K
$\beta_{i1}$ = Beta of Return of Asset i to Factor K
$\varepsilon_i$ = Error term

It is difficult to ascertain the precise value that is expected by the market. More sophisticated models can be employed, e.g. a Vector Autoregressive Model (VAR). A more straightforward and less robust step is to measure the deviation from the mean value for the factor on a conditional or unconditional basis.

## Global Factors

The global factors that are used to determine the return and risk drivers derive from economic or markets indicators. In this example, the model focuses on the US investor. The factors are divided into groups to capture the different dimensions of the US financial markets and economy.

**Global Equity Markets** – This measure the equity component of the global markets and is a substitute for the equity risk premium. The index used is the S&P 1200 Global Index (SPG), which captures about 80% of the total market capitalization of the global equity markets.

**US Inflation** – This measure the inflation impact on the global economy. The inflation rate is constructed from the consumer prices indices (CPI) from our collection of global economies and is on an equally weighted basis.

**US Long Bond Yields** – This is benchmark cost of funding for long-term investments. The Long Government Bond Yield (LGB) rate is constructed from the 10-year yields on Government Bonds from our collection of global economies and is on an equally weighted basis.

**US Short Bond Yields** – This is benchmark cost of funding for short-term investments and is our proxy for cash investments (the risk-free rate). The Short Government Bond Yield (SGB) rate is constructed from the 3-month yields on Government Bonds from our collection of global economies and is on an equally weighted basis.

**Foreign Exchange** – This is based on the trade-weighted dollar (TWD) for the US against its major trading partners and is in US dollar terms. A proxy for a global currency can also be the Special Drawing Rights (SDR) which is a trade-weighted exchange rate for the G-7 countries and is produced by the World Bank.

**Commodity Prices** – Changes in commodity prices impacts business by changing input prices. They can lead to decreasing (increasing) profitability of the firm unless they raise (reduce) prices to the consumer (flow-through inflation) or increase productivity (expansion in potential GDP). The S&P/Goldman Sachs Commodity Index (GSCI) measures commodity price changes US dollar terms.

**Crude Oil** – Energy prices are crucial both to the business and consumer sectors of the economy. Crude Oil captures the change in the price of energy at the root level. The near-month crude oil future contract measures the price change in Crude Oil (Oil).

While these factors aim to cover all the relevant factors for a US investor, there are still other factors that may impact the markets. This list of factors is indicative of the likely factors and will capture most of the relevant factors in the global markets and are a good starting point for measuring factor exposures for a US investor.

One issue with these factors is that they are to some degree interrelated. Individually the factors act upon the equity markets, and Crude Oil is a component of the GSCI. Before the factors enter the equation, a multi-step process isolates the factors:[2]

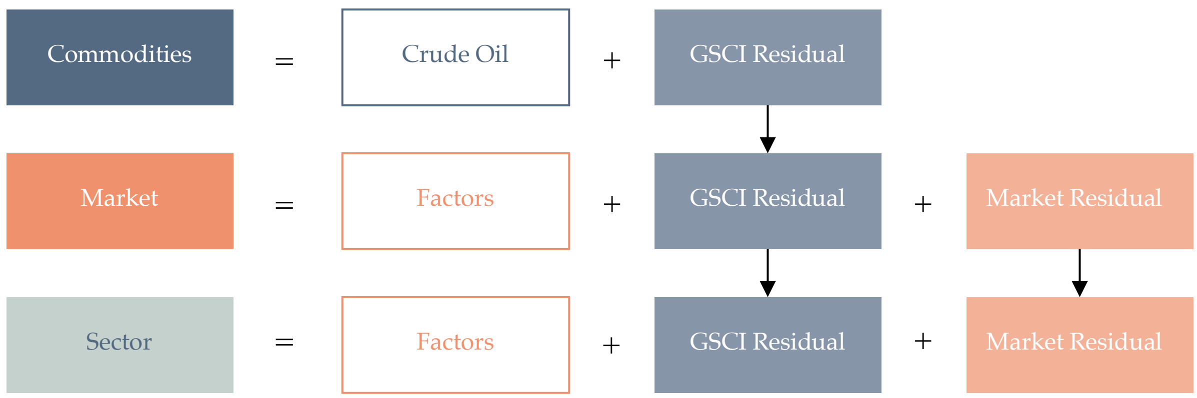
1. A regression determines how much of the variation of the GSCI (dependent) is explained by Crude Oil (independent).
2. The residuals of the prior regression are the commodity factor
3. The returns of the market factor (dependent) are regressed against all the other factors (independent) and the residual is used as the market factor.

[2] The goal of the process is to ensure that each of the factors are orthogonal, i.e. uncorrelated to each other, so that the models are not bias.

4. The residual is then used as a proxy in the regression for the equity factor as it is the proportion of the equity factor that is *not* explained by the other factors.

This process helps to ensure that we reduce the chance of cross terms explaining the variation in the return series. For example, in the case of Oil it is a component of the GSCI, and we want to focus on the proportion of the GSCI that is *not* explained by the Oil component of the GSCI.

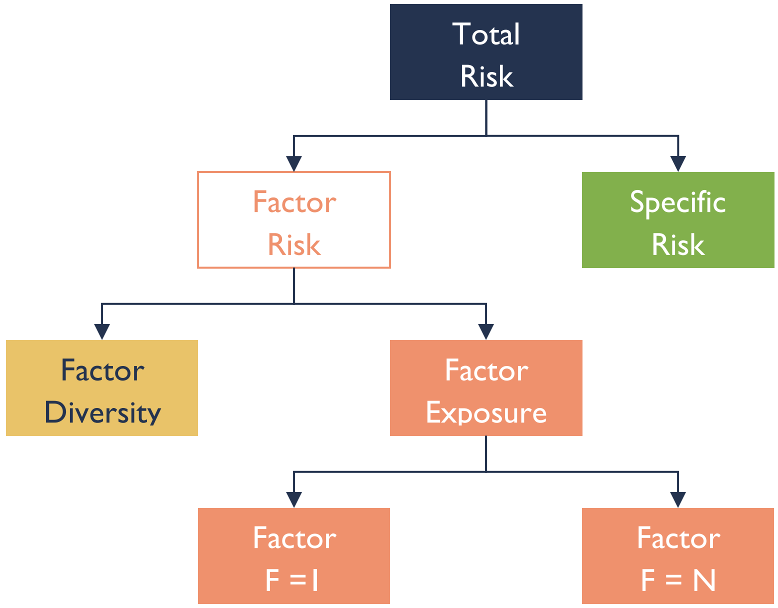
Figure 2. Isolating Market and Oil Factors




## Risk Decomposition

The other side of the coin is to measure the portfolio risk from the factor exposures. In a Capital Asset Pricing Model (CAPM) there are three sources of risk in a portfolio: the market exposure (systematic/undiversifiable), the factor sensitivities and the specific (diversifiable). The particular risks of the assets are viewed to have cross-correlations of zero to reflect the idiosyncratic risk. Figure 3 shows a schematic of the decomposition.

Figure 3. Factor Risk Decomposition




This specification gives the total risk of the portfolio exposure to the risk factors and the specific risk of the portfolio. This specification may be higher than the actual portfolio risk because it does not take into account the cross-correlations that occur between the factors and act to reduce risk.

An adjustment is made to account for the risk reduction. This adjustment is analogous to the diversification benefit of a portfolio, and the expression will look familiar to those that employ portfolio optimization.

The final step is to measure the proportion of the risk from each factor. This step takes the factor risk and proportions it to the specific risk factors. This final step is vital because it provides the last piece required to optimize a portfolio on a risk-adjusted basis for a particular factor.

## Application of Factor Profiles

The process provides an analysis of the relationship between the markets and the global factors. This process can aid the investment decision process for the portfolio in numerous ways, including:

**Factor Analysis** – The relationship between the economic factors and the markets provides insight into the drivers of return and volatility, which provides a means to allocate within the portfolio efficiently.

**Diversification** – A portfolio can be well diversified in the context of Mean-Variance Optimization (MVO), but may not be regarding a specific risk factor (e.g. Oil). This process provides a means to measure the exposure and diversify the portfolio with regards to the risk factor(s).

**Alpha Generation** – The various markets may have cross-sectional differences in their exposures to a given factor, and this process can be used to generate portfolio alpha by tilting the portfolio in a manner that maximizes the relative value to a specific factor.

**Risk Management** – The portfolio exposures to a given factor can be managed by reducing the risk exposure to a given factor. If the portfolio had exposure to Oil, and the exposure was not deemed prudent, the portfolio composition can be altered to reduce the exposure to the factor.

When reviewing the factor profiles, we expect a few related outcomes to occur. In general, the factor coefficients from the multi-factor regression (MFR) will be in the same direction but of different magnitude for the three primary return generating factors (Equity, Yields, and the Yield Curve).[3] The remaining risk factors are expected to vary in size and direction to reflect their varying exposures to the risk factors.

## Process Design

The process for defining the exposures of the global factors can be broken down into four distinct steps:

**Data Collection** – In this stage, the data is sourced from Bloomberg for the market and economic data within the defined range and given frequency.

**Regression Analysis** – A two-step process starts by creating the residuals for the equity market and the commodity exposures.

1. Regress the commodity returns against the crude oil returns and calculate the time-series of the residual from the regression. The result is the proportion of commodities that is *not* explained by crude oil.
2. Regress the global equity returns against the returns of all the other factors. Calculate the time-series of the residual from the regression,

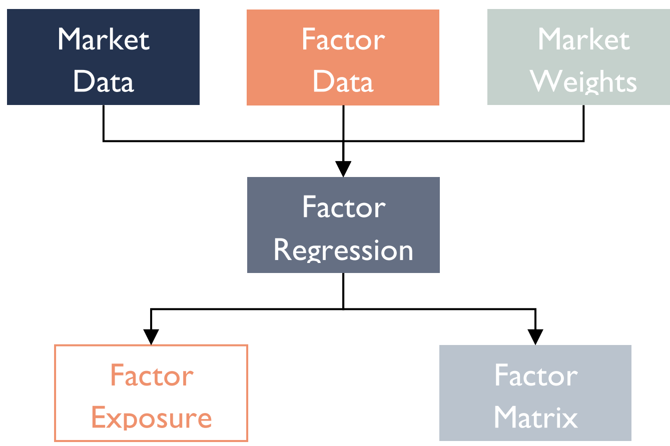
[3] The technical assumption is that these are expected to be non-zero and concurrently not equal to the same value. A Wald test can be undertaken to determine if the statistical outcome is valid.

which results in the proportion of global equity that is *not* explained by the other factors.

**Return Summary** – Organize the factors to display the relevant coefficients (betas) with the degree of explanatory ability highlighted by the error of the equation. Define the confidence interval for the coefficients (e.g. 90% level).

**Risk Summary** – Organize and calculate the proportion of variation in each of the individual markets that are explained by the factors. Factors may contribute to the total risk or may reduce the risk depending upon their correlation between the factors.

Figure 4. Multi-Factor Model Process Flow




These steps provide a brief analysis of the return and risk profile of the markets and may augment the investment decision process for the investment portfolio.

## Portfolio Factor Tilts

The process delivered answers to two critical questions: the factor returns sensitivity and the decomposition of the risk. While these are helpful measures, the *management* of the portfolio is the most crucial aspect of the process. As in traditional MVO analysis, the goal is to create *efficient* portfolios while at the same time expressing a view. With only one factor, it is merely a question of how much market exposure the investor wants to achieve: a higher beta would provide more market exposure, while a lower beta would deliver less market exposure. The process is similar with multi-factors but requires a few more constraints during the optimization process.

A portfolio optimization process seeks to maximize (minimize) the beta exposure to the selected factor (e.g. US dollar). Without constraints, the process is free to vary weights, total risk, risk composition, and tracking error versus the benchmark. This outcome is acceptable in a world of perfect certainty of a forecast. Unfortunately, most investors do not possess perfect insight, and thus the inputs into the process are critical since they can materially skew the portfolio optimization process (Michaud, 1989).

To manage the risk from expressing a view that is less than certain, an investor can place constraints on the process. In this way, the investor "aims big and miss small." For investors with benchmarks, the constraints will follow the following form:

- Individual asset (classes) weights have maximums and minimums
- Expected total portfolio risk is equal to or less than the benchmark
- Expected tracking error versus the benchmark is limited (e.g. 2%)
- Risk contribution from a specific factor (e.g. equity risk <= 50%)

Other constraints are applied as required (e.g. non-negative asset weights) by the investment policy statement.

It is critical to understand that when expressing a view in a multi-factor model, the investor is explicitly making a call on the *direction* of the specific factor. If the view is not as expected, then the portfolio may endure underperformance versus the benchmark due to a lower return, even though the view is expressed as efficiently as possible. Fortunately, minimizing the risk when the outcome is different than expected still adds value.

## Appendix A. Risk Decomposition

The formulaic expression is as follows:

$$V_P = w'\beta V_X \beta' w + w'V_\varepsilon w$$

Where $V_P$ = Portfolio Variance
$w$ = Vector of Portfolio Weights
$V_x$ = Covariance Matrix of Risk Factor Returns
$\beta$ = Vector of Equity Betas
$V_\varepsilon$ = Specific Risk Covariance Matrix

This specification gives the total risk of the portfolio exposure to the risk factors and the specific risk of the portfolio. This may be greater than actual portfolio risk because it does not take into account the cross correlations that occur between the factors.

To account for the risk reduction, a final adjustment is done to take it into account. This is analogous to the diversification benefit of a portfolio, and the expression will look familiar to those that employ portfolio optimization.

$$V_P = \underbrace{w'\beta V_X \beta' w}_{\text{Factor}} + \underbrace{w'V_\varepsilon w}_{\text{Specific}} - \underbrace{2(1-\rho)\sigma_x^2\sigma_\varepsilon^2}_{\text{Covariance}}$$

Where $V_P$ = Portfolio Variance
$w$ = Vector of Portfolio Weights
$V_x$ = Covariance Matrix of Risk Factor Returns
$\beta$ = Vector of Equity Betas
$V_\varepsilon$ = Specific Risk Covariance Matrix
$\rho_{x,\varepsilon}$ = Correlation of Factor Risk and Residual Risk
$\sigma_{x,\varepsilon}$ = Standard Deviation of Factor and Residual Risk

In order to isolate the Factor Risk profile, a further calculation is undertaken that apportions the factor risk to the individual factors.[4] The following calculation returns a vector of the risk contribution from each factor:

$$V_F = \frac{\beta'\beta' V_X}{\beta' V_X \beta} \times \beta' V_X \beta$$

[4] For a discussion of risk decomposition see the MSCI Barra Risk Model Handbook, Version 3 US, pp-23-29. Further details are in Active Portfolio Management by Grinold and Kahn, 2000.

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**Capital Risk Management LLC**
415-373-7152
contact@capitalriskmanagement.com

www.capitalriskmanagement.com
San Francisco | Toronto

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